Here we study large deviation functions (LDF) of time-integrated current for a number of variants of TASEP by adapting the iterative Measurement-and-Feedback method [1]. Firstly, we study LDF for TASEP on a ring, both analytically and numerically to justify this method. We compare our exact results using Bethe ansatz with simulations based on the iterative method, analyzing order of corrections in this method and derive conditions of its applicability. Then, we adapt the existing methods to the setting of discrete time Markov chains to study LDF for discrete time TASEP with open boundaries as a minimal stochastic traffic model.

Bethe ansatz for TASEP on a ring for $s < 0$

Each configuration of the system is represented by a strictly increasing sequence of integers of length $N$ which determine the positions of $N$ particles, i.e., $C = \{n_1, n_2, \ldots, n_N\}$. The eigenfunction of a configuration $C$ is written as

$$\psi(n_1, n_2, \ldots, n_N) = \sum_{\pi \in S_N} A_{\pi(n_1)} A_{\pi(n_2)} \cdots A_{\pi(n_N)}$$

where $S_N$ is a set of all permutations of the integers $1, 2, \ldots, N$. The Bethe equations are

$$\zeta_k = \sum_{i \neq k} \frac{e^{\epsilon_i - \epsilon_k}}{e^{\epsilon_i} - 1} \quad \text{for } k = 1, 2, \ldots, N$$

For any solution $\{z_k\}$, (2) gives an eigenvector of matrix $U$ with eigenvalue $\lambda = e^{\epsilon(1/N)} - 1$. From periodicity condition $A_{z_1, z_2, \ldots, z_N} = A_{z_2, z_3, \ldots, z_N, z_1}$, (3) holds.

Our starting point is the results in [3] where the authors estimated $\{z_k\}$ for which $\lambda$ is maximized. It is the case where $N - 1 \leq z_k \leq e^\epsilon$, and one is $e^{\epsilon(1/N)}$. (Here we assume $z_1$ is this one).

- The non-zero terms in equation (2) are those in which the amplitudes can be written in terms of the amplitude of the identity permutation, using the periodicity condition.
- There are $N$ nonzero terms; all transpositions of 1 with other integers.
- By substitution we obtain

$$\psi(n_1, n_2, \ldots, n_N) = \sum_{j=1}^N e^{\epsilon(j-1)} \sum_{u=1}^N \sum_{s=N+e(j-1)} e^{\epsilon(N+e(j-1))}$$

- The equation (5) is translational invariance.
- $\psi(n_1, n_2, \ldots, n_N)$ attains minimums at equidistant configurations, and maximum at configurations with one cluster.

Error of the estimation

To evaluate this, we calculate the ratio between the largest and second largest values of $\psi(C)$. The latter attains in configurations with two clusters, one is of size $N - 1$, and the other one is a single particle cluster. $\epsilon$ is the distance between the clusters.

$$\frac{\psi(n_1, n_2, \ldots, N-1, n_{N-1}, \ldots, n_N)}{\psi(n_1, n_2, \ldots, N-1, \ldots, n_N)} = e^{\epsilon} + O(e^{\epsilon(N-1)/(2-N)})$$

References