Stochastic Quantum Zeno by Large Deviation Theory

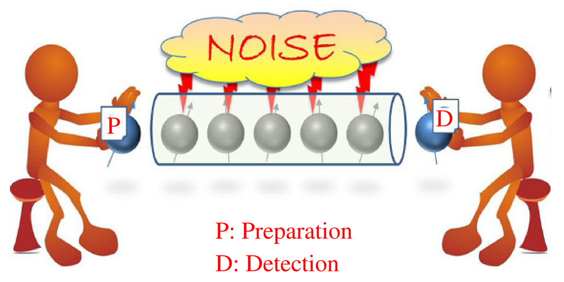
Shamik Gupta
Max Planck Institute for the Physics of Complex Systems, Dresden, Germany

Abstract

Quantum measurements are crucial for observing the properties of a quantum system, which, however, unavoidably perturb the dynamics of the system in an irreversible way. Here we study the dynamics of a quantum system being subjected to a sequence of projective measurements applied at random times. In the case of independent and identically distributed intervals of time between consecutive measurements, we analytically demonstrate that the survival probability of the system to remain in the projected state assumes a large deviation (exponentially decaying) form in the limit of an infinite number of measurements. This allows us to estimate the typical value of the survival probability, which can therefore be tuned by controlling the probability distribution of the random time intervals. Our analytical results are corroborated by both numerical and experimental results. Our studies provide a new tool for protecting and controlling the amount of quantum coherence in open quantum systems by means of tunable stochastic measurements.

Quantum System Subject To Projective Measurements At Random Times

Modeling an open quantum system interacting with the external environment [1]:



A quantum system subject to a sequence of projective measurements at random times

- **Initial state** $|\psi(t_0)\rangle \equiv |\psi_0\rangle$
- **Unitary evolution for random time** μ_1**: Evolved state** $|\psi(t_0+\mu_1)\rangle = \exp(-iH\mu_1/\hbar)|\psi_0\rangle$ (H **: System Hamiltonian**)
- **Projective measurement to the initial state: Projection operator** $\Pi \equiv |\psi_0\rangle\langle\psi_0|$
- **State after the measurement:** $\Pi|\psi(t_0+\mu_1)\rangle$
- **Further evolution for random time** μ_2**, followed by a projective measurement, and so on**
- **Consider an arbitrary but fixed number** m **of consecutive measurements separated by *random* time intervals** μ_j**, with** $j=1,\ldots,m$**; Measurement sequence:** $\{\mu_j\}$
- **Survival probability** $\mathcal{P}(\{\mu_j\})$ **to be in the initial state after** m **measurements, for realization** $\{\mu_j\}$**:**

Random time with distribution $p(\mu)$

$$\mathcal{P}(\{\mu_j\}) = \prod_{j=1}^{m} q(\mu_j)$$

Measurements

$$q(\mu_j) \equiv \left|\langle\psi_0|\exp(-iH\mu_j/\hbar)|\psi_0\rangle\right|^2$$

$\mathcal{P}(\{\mu_j\})$ **is a random variable with respect to different realizations of** $\{\mu_j\}$

- **Expect:**

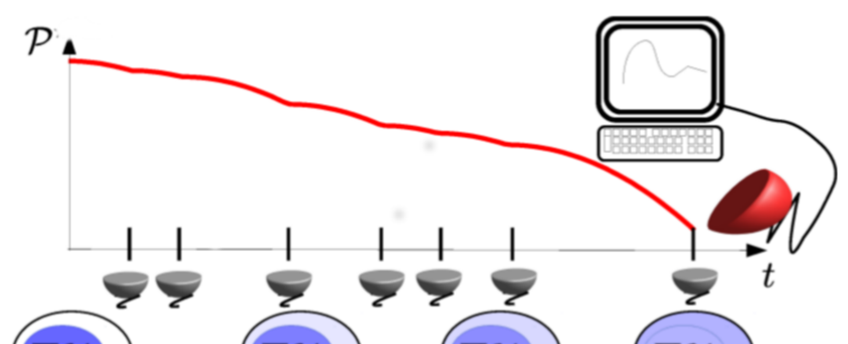


Figure 1: Decay of the survival probability $\mathcal{P}$ for a quantum system to remain in Hilbert subspace $\mathcal{H}$ when subjected to a stochastic sequence of measurements. As time goes on, the population leaks out of the subspace ($\Pi\mathcal{H}$), as shown by the blue shades. Only the final survival probability is registered by a detector marked in red [2]

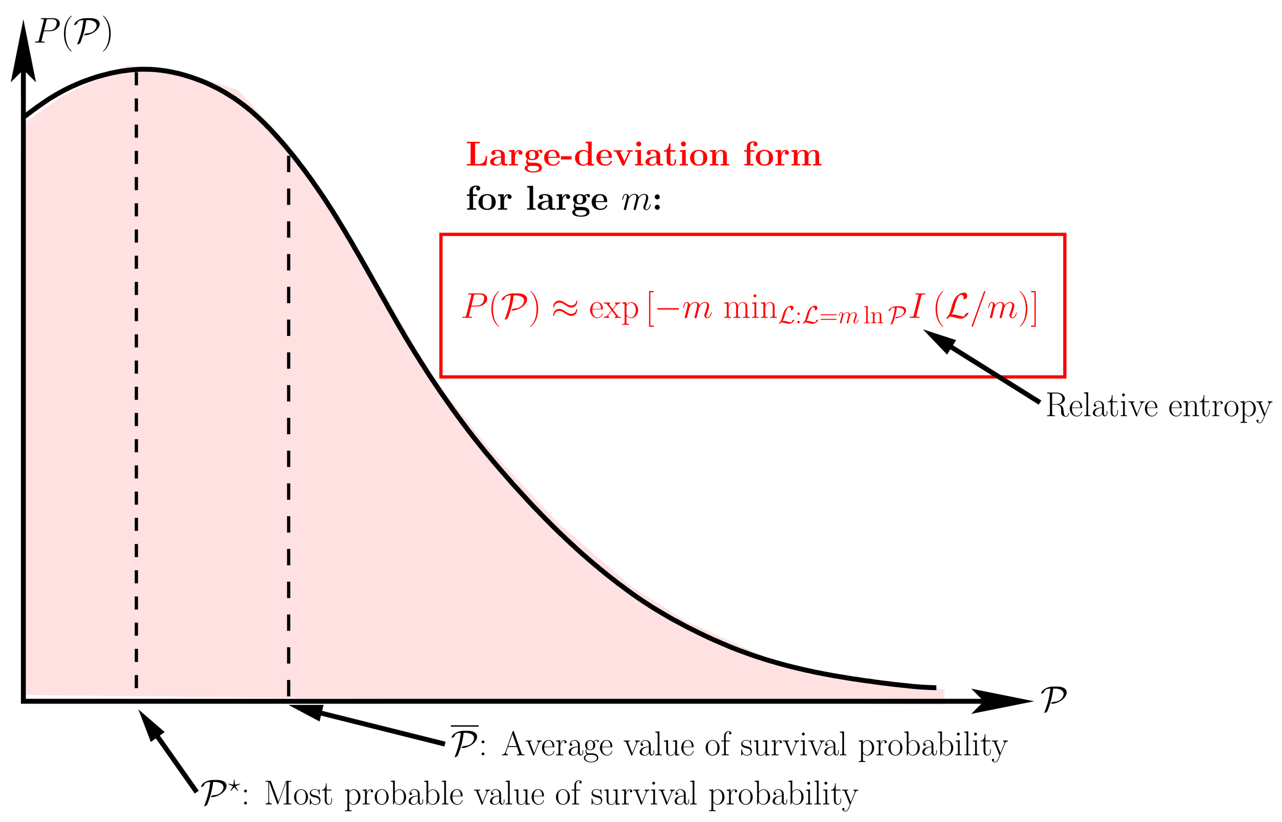
Survival Probability And Large Deviations [3]

Questions:

- **Is it possible to have realizations of measurement sequence that give values of the survival probability significantly deviated from the mean (Large deviations) ?**
- **How typical/atypical are those realizations ?**

Answer(s):

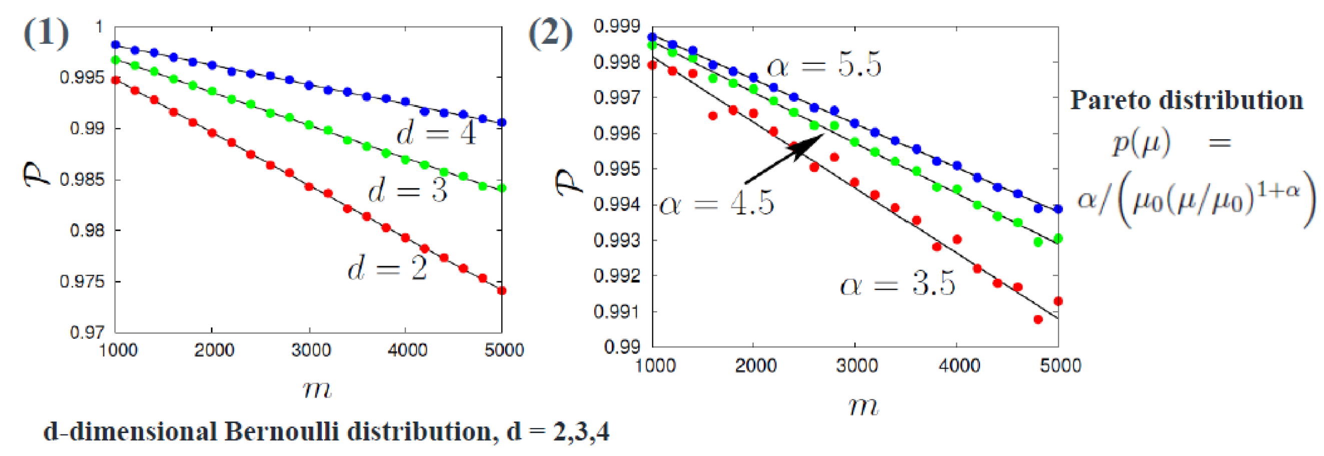
- **In case of independent and identically-distributed random intervals of time between measurements, the survival probability** $\mathcal{P}$ **assumes a Large Deviation (exponentially decaying) form in the limit of an infinite number of measurements** ($m\to\infty$)



- **For a** d**-dimensional Bernoulli distribution** $\{p^{(\alpha)}\}_{\alpha=1,2,\ldots,d}$**:**
  $I(x) = \sum_{\alpha=1}^{d} f(\mu^{(\alpha)}) \ln\left(\frac{f(\mu^{(\alpha)})}{p^{(\alpha)}}\right)$; $f(\mu^{(\alpha)}) = \frac{\ln q(\mu^{(d)})-x}{(d-1)\left[\ln q(\mu^{(d)})-\ln q(\mu^{(\alpha)})\right]}$; $\alpha=1,\ldots,(d-1)$, $f(\mu^{(d)}) = 1-\sum_{\alpha=1}^{d-1} f(\mu^{(\alpha)})$
- $\mathcal{P}^{\star} = \exp\left(m\int d\mu\, p(\mu)\ln q(\mu)\right)$; $\overline{\mathcal{P}} = \exp\left(m \ln \int d\mu\, p(\mu) q(\mu)\right)$
- $\langle \exp(x)\rangle \geq \exp(\langle x\rangle) \Rightarrow \overline{\mathcal{P}} \geq \mathcal{P}^{\star}$

Numerical Demonstration [3]

- **Generic** n**-level quantum system:** $H = \sum_{j=1}^{n}\omega_j|j\rangle\langle j| + \sum_{j=1}^{n-1}\Omega\left(|j\rangle\langle j+1| + |j+1\rangle\langle j|\right)$
- $n=3$; $|\psi_0\rangle \equiv \frac{1}{\sqrt{2}}(|100\rangle + |001\rangle)$



Solid lines: Theory for $\mathcal{P}^{\star}$**, Points: Results for a *typical* realization of measurement sequence**

Experimental Demonstration [4]

- **Set up: Bose-Einstein-condensed** ^{87}**Rb atoms produced in a magnetic micro-trap**

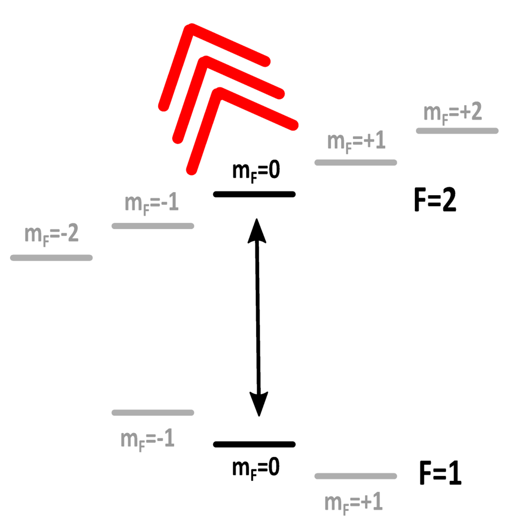
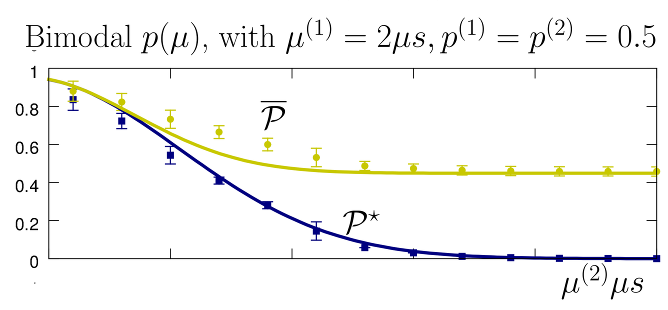


Figure 2: Ground state of ^{87}Rb atoms in a magnetic field has two hyperfine levels $F=1$ and $F=2$. A laser-induced Raman transition couples the sub-levels $|F=1, m_F=0\rangle$ and $|F=2, m_F=0\rangle$, while a laser resonant with the transition $|F=2\rangle \to |F'=3\rangle$ (red arrows) depletes the population of $|F=2\rangle$, leading to an effective projective measurement



Solid lines: Theory, Points: Experiment

Quantum Zeno Effect [5]

- m **measurements equally spaced by** Δt**:** $m\Delta t = t$
- **Limit of infinitely frequent measurements in a fixed time** t**:**
  Survival probability: $\mathcal{P} = \overline{\mathcal{P}} = \mathcal{P}^{\star} = \lim_{m\to\infty}[q(\Delta t)]^m = 1-(t^2/m)\left(\langle H^2\rangle - \langle H\rangle^2\right) \approx 1$

A watched kettle does not boil !!

Stochastic Quantum Zeno Effect [3]

- $q(\mu) = 1-\mu^2\left(\langle H^2\rangle - \langle H\rangle^2\right)$
- **As** $m\to\infty$**:**
  $\mathcal{P}^{\star} = \overline{\mathcal{P}} \approx \exp\left(-m\int d\mu\, p(\mu)\mu^2\right)$
- $\mathcal{P}^{\star} = \overline{\mathcal{P}} \approx 1$, provided $\int d\mu\, p(\mu)\mu^2 \ll 1$ **(Condition to obtain Stochastic Quantum Zeno Effect)**

Applications

- **Novelty of the measurement scheme: Easy to control the number of measurements** m **while letting the total duration of observation** $\sum_{j=1}^{m}\mu_j$ **fluctuate between realizations of** $\{\mu_j\}$
- **Protection against decoherence: Most probable survival probability of a state tunable by choosing suitable** $p(\mu)$**; in a typical dynamical evolution, the state can thus be made to survive for desired duration**

References

[1] F. Caruso, V. Giovannetti, C. Lupo, S. Mancini, Rev. Mod. Phys. **86**, 1203 (2014)
[2] M. M. Müller, S. Gherardini, A. Smerzi, F. Caruso, arXiv:1606.05369
[3] S. Gherardini, S. Gupta, F. S. Cataliotti, A. Smerzi, F. Caruso, S. Ruffo, New J. Phys. **18**, 013048 (2016)
[4] S. Gherardini, C. Lovecchio, M. M. Müller, P. Lombardi, F. Caruso, F. S. Cataliotti, arXiv:1604.08518
[5] B. Misra and E. C. G. Sudarshan, J. Math. Phys. **18**, 756 (1977)

Acknowledgements:

www.qdab.org

Contact: shamikg1@gmail.com