Emergence of cooperation: Growing habitat with empty sites

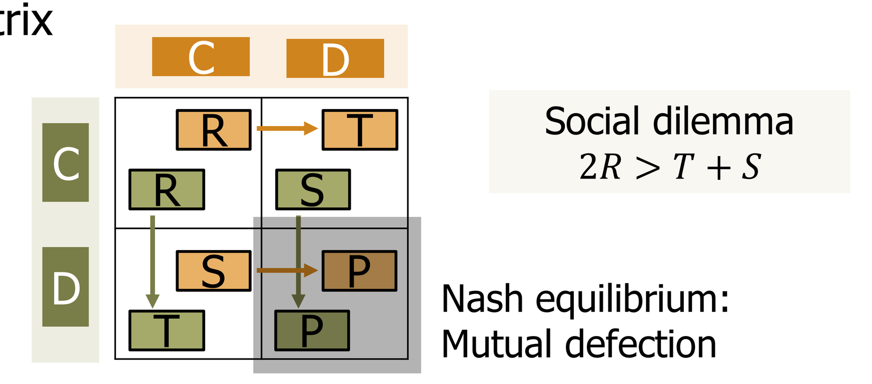
Hye Jin Park[1], Christian Hilbe[2,3], Martin A. Nowak[3], Beom Jun Kim[1], and Hyeong-Chai Jeong[4]

[1] *Department of Physics, Sungkyunkwan University, Suwon 16419, Korea*
[2] *Institute of Science and Technology Austria, Am Campus 1, Klosterneuburg 3400, Austria*
[3] *Program for Evolutionary Dynamics, Harvard University, Cambridge, MA 02138, USA*
[4] *Department of Physics, Sejong University, Seoul 05006, Korea*

Motivation

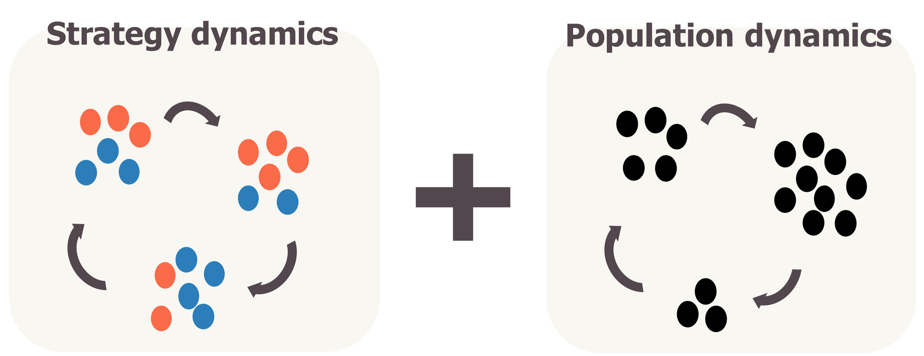
Prisoner's dilemma game

- Possible action: Cooperation (C) and Defection (D)
- Payoff matrix



$T > R > P > S$

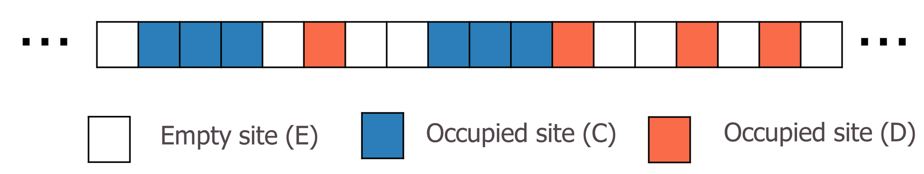
How does cooperation emerge?



Combining strategy dynamics and population dynamics

Model

Birth and death processes with an infinite 1-dimensional lattice



Death : an individual who lives in an occupied site dies with death probability.

Birth : an empty site is occupied by neighboring agent's offspring.

- **Algorithm**
  1. Select a site i at random
  2. If $S_i = C, or\ D$: Death process
     $S_i = E$: Birth process
  3. Iterate 1-2

- **Death probability**

$$\frac{1}{1 + Ae^{\omega p_i}}$$

p_i : total payoff
ω : selectivity ($\omega = 1$)
A : fitness parameter (>0)

- **Simplified PDG payoff matrix**

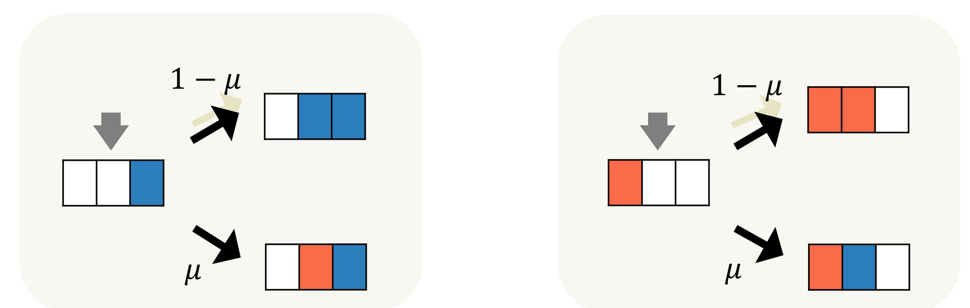
$$M = \begin{array}{c|ccc} & C & D & E \\ \hline C & b-c & -c & 0 \\ D & b & 0 & 0 \end{array}$$

$0 < c < b$
$b = 1$

- **i-th site's total payoff**

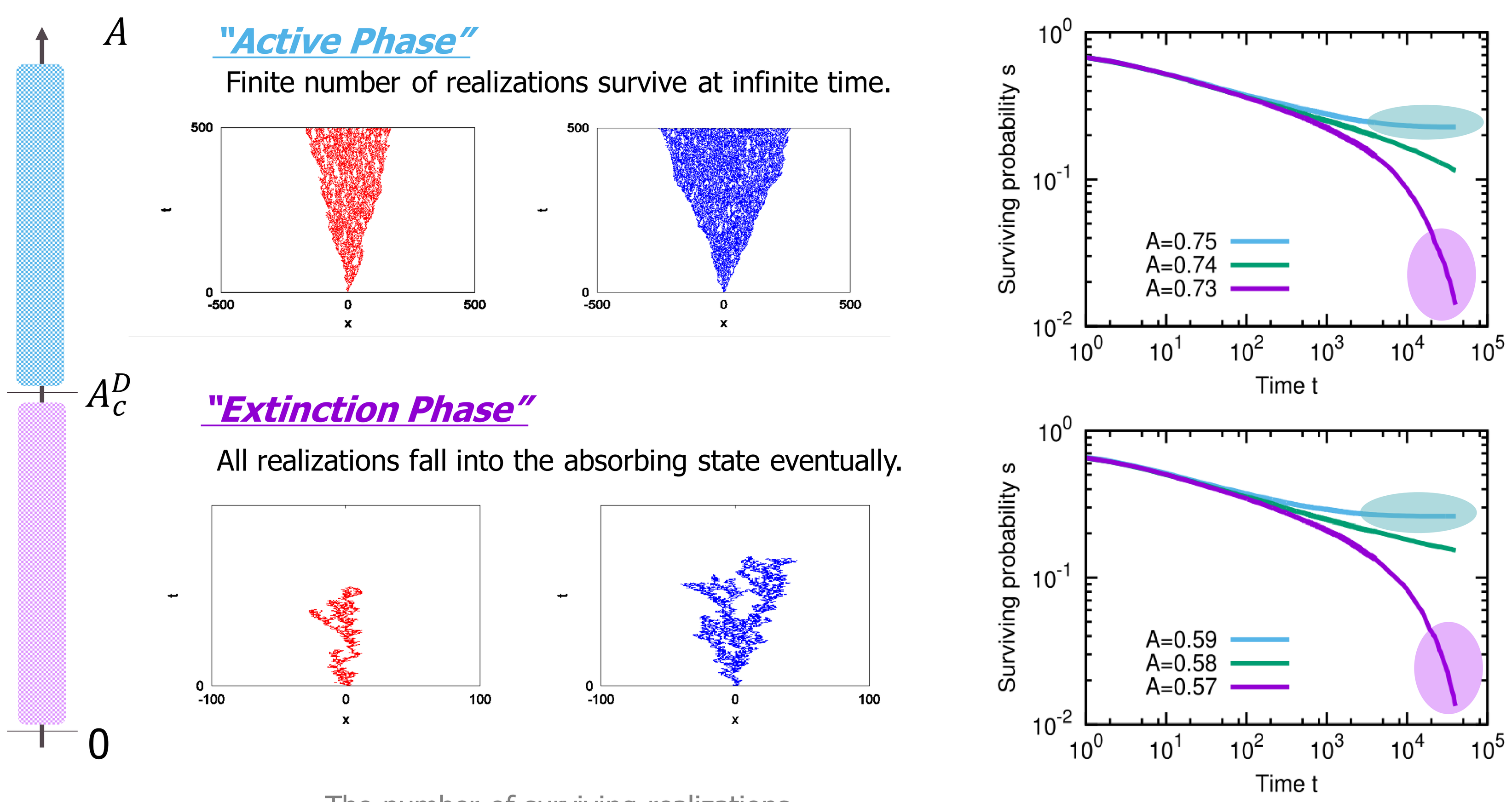
$$p_i = M_{s_i s_{i-1}} + M_{s_i s_{i+1}},\ s_i \in \{C, D, E\}$$

- **Mutation μ**



Without mutation

Absorbing transition points A_c



$$s(t) = \frac{\text{The number of surviving realizations}}{\text{The total number of realizations}}$$

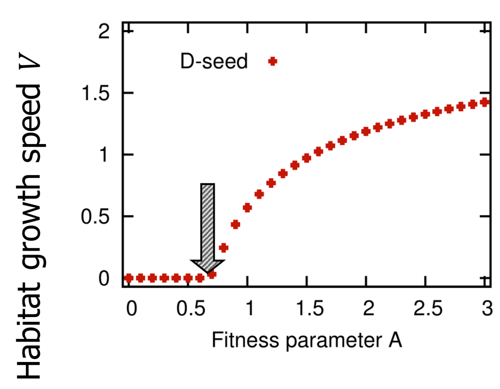
Mean-field calculation

$$V_D = 2\left[1 - \alpha\rho \sum_{n=1} n(1-\rho)^{n-1}\right] = 2\left(1 - \frac{\alpha}{\rho}\right)$$

ρ: population density α: death probability

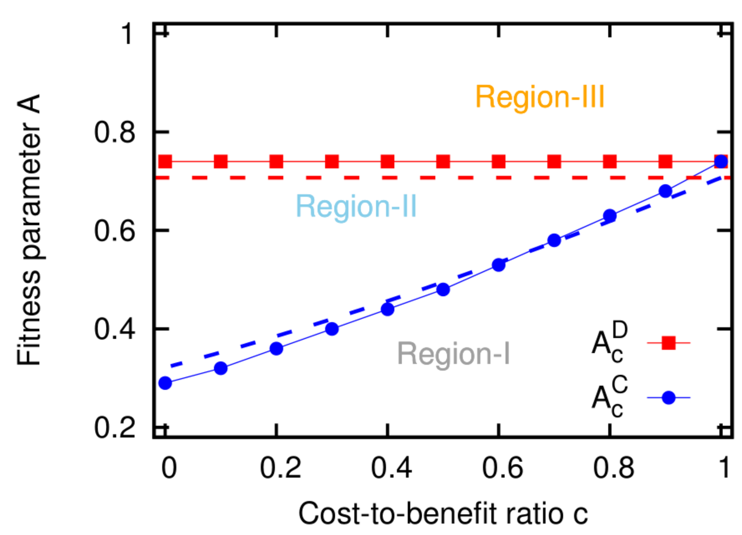
$$\frac{d\rho}{dt} = (1-\rho)(2\rho - \rho^2) - \alpha\rho$$
$$= 0 \text{ (at steady state)}$$

$$V_D = 2\left[1 - \frac{2}{3(A+1) - \sqrt{(A+1)(A+5)}}\right]$$



$V = 0$ **at** A_c^D

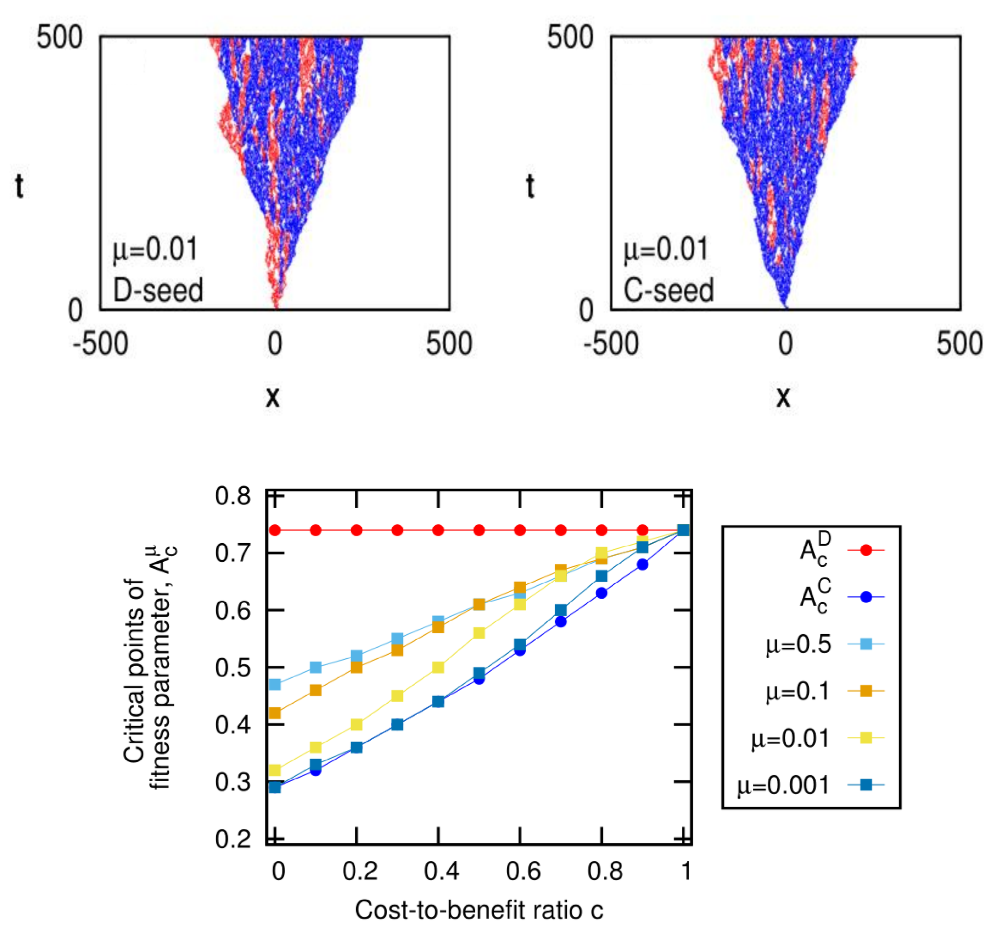
Phase diagram



In the region-II, only the cooperators can survive.

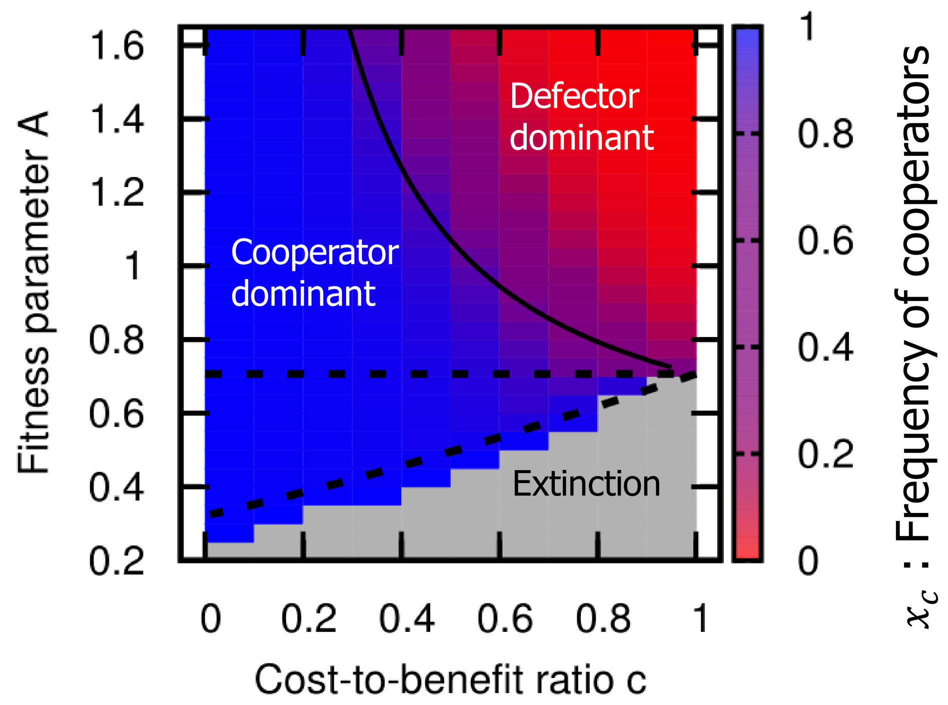
With mutation

Absorbing transition points A_c^μ



Initial condition dependency disappears.

Phase diagram ($\mu = 0.001$)



Empty sites protect the cooperators from the invasion of defectors, and cooperators can survive even though $c/b > 1/k$.

Summary and discussion

- We have considered the population dynamics as well as the strategy dynamics by introducing empty sites.
- We have found that the empty sites created from population dynamics construct spatial structure and develop the cooperative society.
- Our model in 2-dimensional lattice: future research.

jina88@skku.ac.kr