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Prediction and control of slip-free rotation states in sphere assemblies

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Balls as 3D Gears
APS Physics Focus Story

D. V. Stäger[1], N. A. M. Araújo[2], H. J. Herrmann[1,3]
[1]**Computational Physics for Engineering Materials, IfB, ETH Zurich;** [2]Departamento de Física,Faculdade de Ciências, and Centro de Física Teórica e Computacional, Universidade de Lisboa; [3]Departamento de Física, Universidade Federal do Ceará

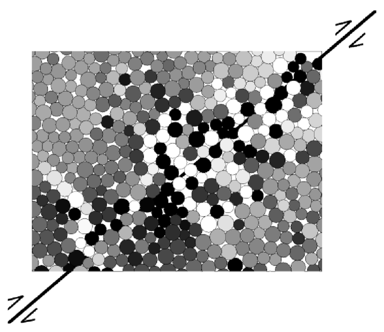
1 Motivation

Background

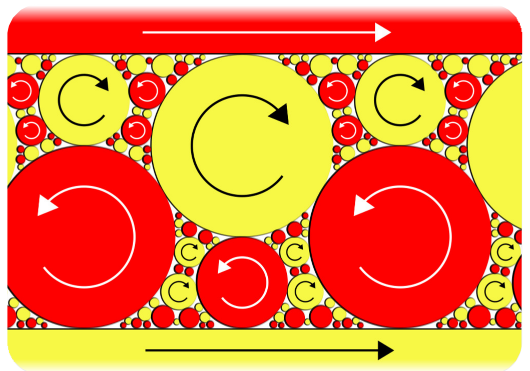
Shear bands in disk packings show formation of **bipartite** assemblies with synchronized rotation

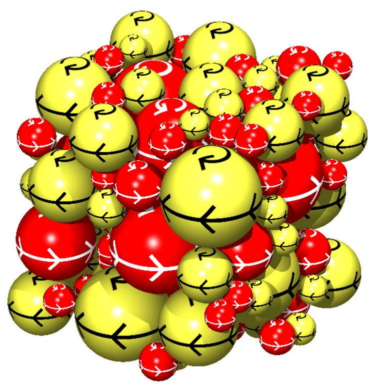
what?

Bipartite assemblies have slip-free rotation states in 2D and 3D [*Phys. Rev. Lett. 92, 044301 (2004)*], and serve as simplified models for tectonic shear zones with unexpected low frictional heat formation



Rotations in gray scale
Phys. Rev. Lett. 84, 638 (2000)



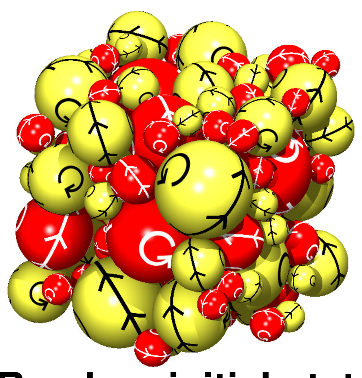


Bipartite assemblies in slip-free rotation states

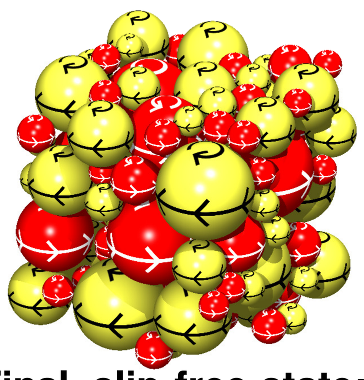
Tectonic shear zone

Driving question: **How do bipartite assemblies synchronize?**

Bipartiteness
Elements can be colored **RED** or **YELLOW** such that no elements of same color touch



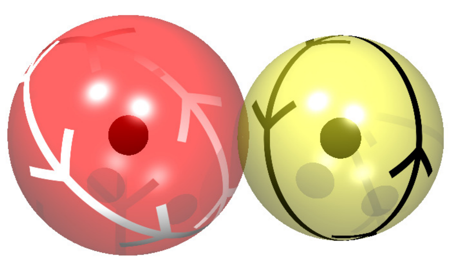
Random initial state: Slip between spheres



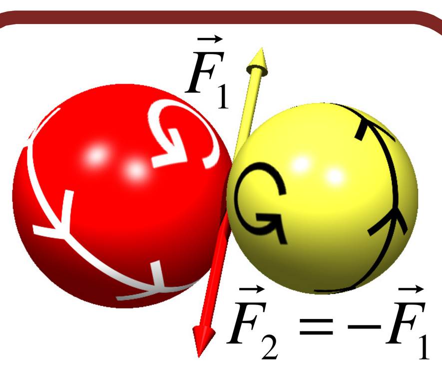
Final, slip-free state: Touching spheres have the same tangential velocity

2 Theory

Model



Spheres fixed at center, but free to rotate



Only sliding friction

Implications

→ Relation between the angular accelerations of contacting spheres

$$m_1 r_1 \vec{\alpha}_1 = m_2 r_2 \vec{\alpha}_2 \quad (\text{simplified for } I_i \propto m_i r_i^2)$$

m = mass, r = radius, $\vec{\alpha}$ = angular acceleration, I = moment of inertia

→ **Time-invariance of special terms of rotational variables:**

2D: one scalar sum | 3D: one vector sum and one scalar sum (3+1)

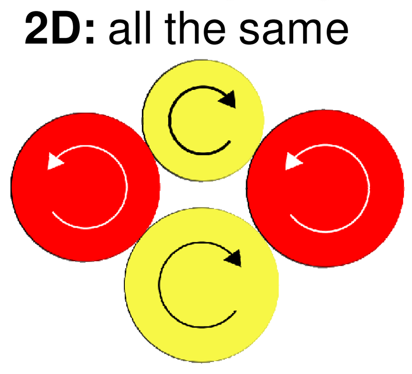
$$A = \sum_i s_i m_i r_i \omega_i \qquad \vec{A} = \sum_i s_i m_i r_i \vec{\omega}_i, \; B = \sum_i s_i m_i r_i \vec{\omega}_i \cdot \vec{x}_i$$

$s_i = +1$, for (yellow) and -1, for (red), $\vec{\omega}$ = angular velocity,
$\vec{x}$ = position from the center of mass of the whole assembly

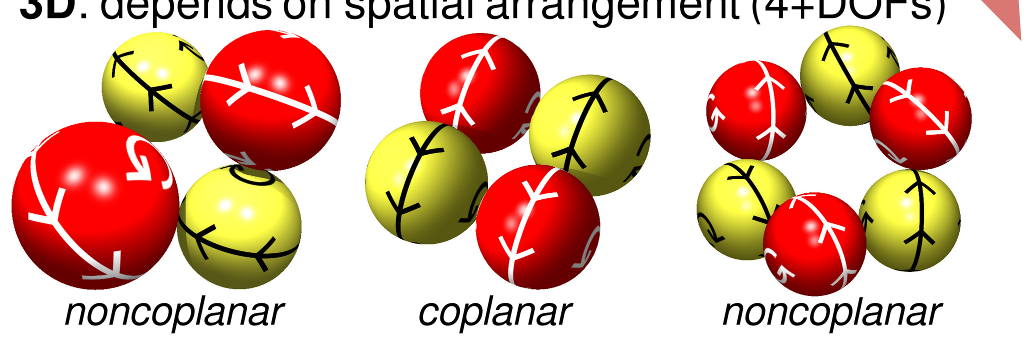
Can we use these to predict the slip-free state?

How many degrees of freedom does the slip-free state have?

2D: all the same | **3D**: depends on spatial arrangement (4+DOFs)



1DOF



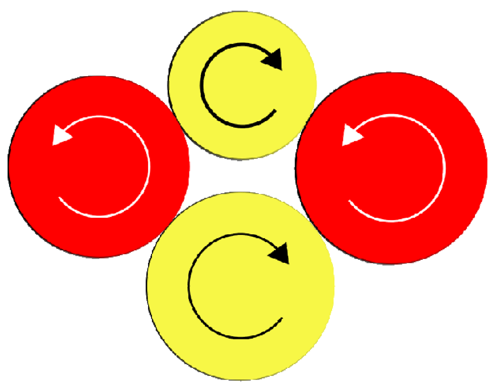
noncoplanar **4DOF** | *coplanar* **5DOF** | *noncoplanar* **6DOF**

3 Results

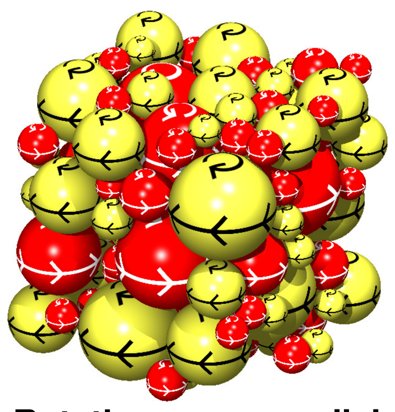
Slip-free state can be predicted by time-invariant terms

In all 2D assemblies | **In 3D assemblies with 4DOFs in slip-free state**

Possible states: | Possible states:

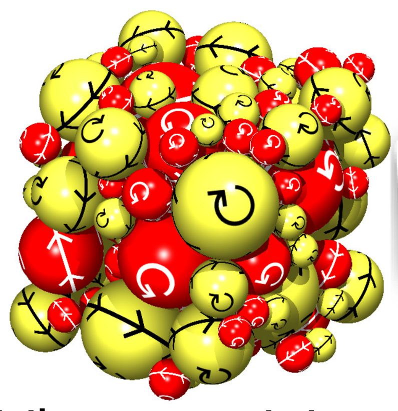


Tangential velocities are equal



Rotation axes parallel (tangential velocities equal)

OR



Rotation axes meet at a point (spheres rotate faster the further away from this point)

These can be constructed systematically!

→ **Final state is independent of the strength of friction**

$$\omega_i^{final} = \frac{s_i A}{r_i M} \qquad \vec{\omega}_i^{final} = \frac{s_i}{r_i}\left(\frac{\vec{A}}{M} + \frac{B}{I}\vec{x}_i\right) \qquad M = \sum_i m_i, \quad I = \sum_i m_i |\vec{x}_i|^2$$

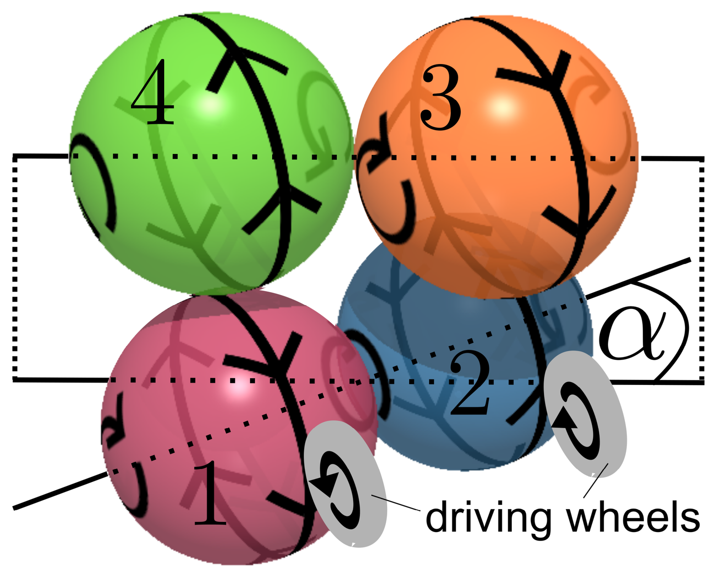
→ **Any slip-free state can be controlled by controlling …**

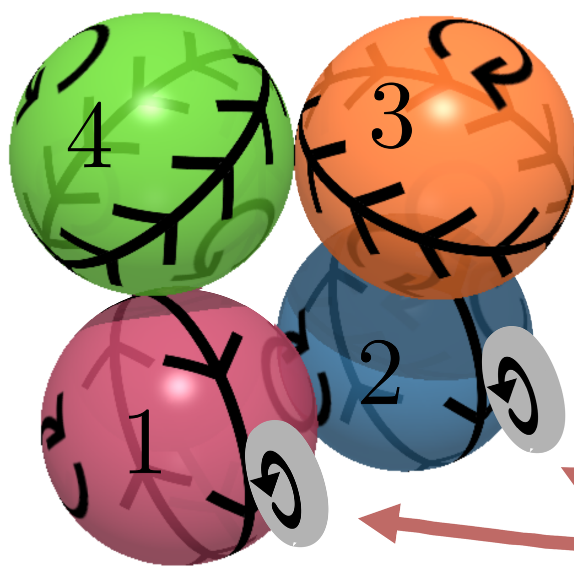
…one disk only | **…two spheres only !**

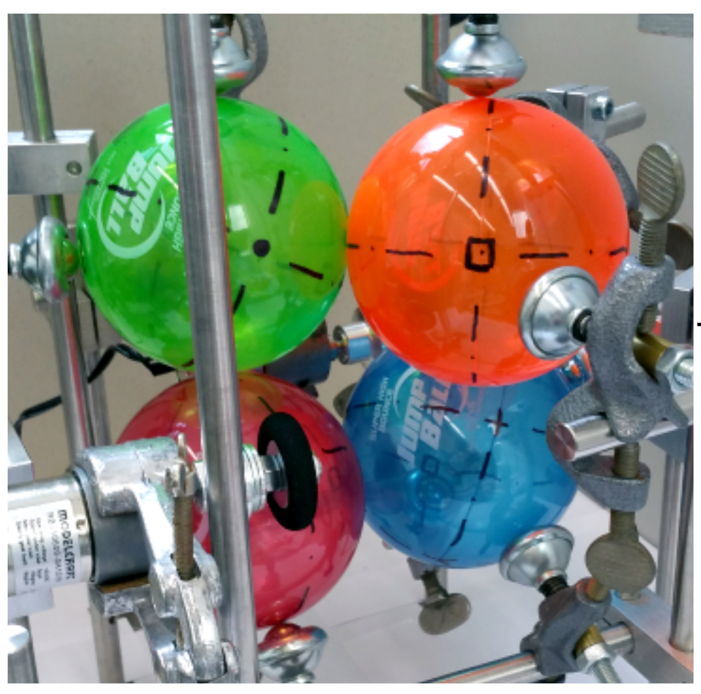
3 Experiment

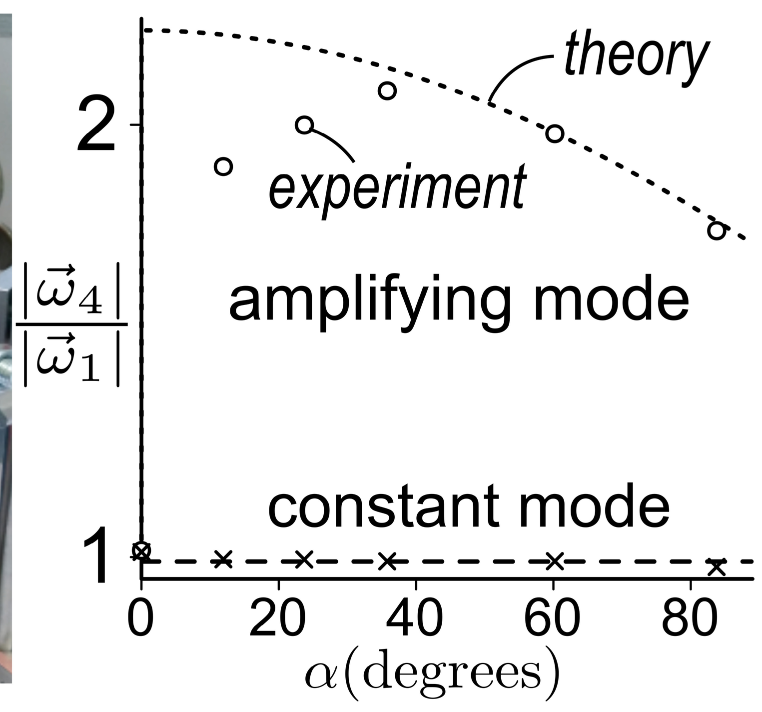
Control of slip-free state

constant mode | amplifying mode









4 Conclusion

- The **ability to control the rotation state** of an assembly of rotating spheres in contact is a **newly discovered functionality** and likely to find use in mechanics and robotics.
- The possibility to **amplify the angular velocities of spheres along an assembly** could be employed as an **alternative to power transmission gears**.